Laboratori Nazionali di Frascati

LNF-63/29 (1963)

R. Gatto: EXPERIMENTS WITH ELECTRON COLLIDING BEAMS.

Estratto dal: Theoretical Physics.

Reprint from

"THEORETICAL PHYSICS"

INTERNATIONAL ATOMIC ENERGY AGENCY
VIENNA 1963

EXPERIMENTS WITH ELECTRON COLLIDING BEAMS

R. GATTO
NATIONAL LABORATORY OF FRASCATI, FRASCATI, ITALY

INTRODUCTION

The two subjects on which I shall talk, $e^+ e^-$ colliding beam experiments and $p\bar{p}$ annihilation into leptons, though pertaining to quite different experimental projects, have a common theoretical background that makes it convenient to discuss them together.

Electron-positron colliding beams constitute the largest project now under development at the Frascati National Laboratory. A first part of the project, the construction of a small ring (called AdA from the Italian Anello di Accumulazione) has now been completed, and the relevant experiments ($e^+ e^-$ annihilations at energies up to ~450 MeV in the centre-of-mass system) are being carried out using for the injection the linear accelerator at Orsay.

The second but more ambitious part of the project consists in the development of one or more larger storage rings, up to energies of ~1.5 GeV for each of the two colliding beams (i.e. 3 GeV total centre-of-mass energy). This second part of the project is called Adone (Adone is the Italian for "Adonis" but can also mean "larger AdA").

The $p\bar{p}$ annihilation experiments are being carried out on the proton-synchrotron of the European Organization for Nuclear Research (CERN) by Conversi, Farley, Müller and Zichichi, using an intense beam of antiprotons.

THE FRASCATI STORAGE RING PROJECT

A description of AdA can be found in a paper presented by TOUSCHEK at the CERN Conference on high energies in 1961 [1], and also in a note by BERNARDINI, CORAZZA, GHIGO and TOUSCHEK [2]. The Adone project is discussed by AMMAN, BERNARDINI, GATTO, GHIGO and TOUSCHEK in a preliminary form [3] and, at a more advanced stage, by AMMAN, BASSETTI, BERNARDINI, CORAZZA, MASSAROTTI, MANGO, PELLEGRINI, PLACIDI, PUGLIGI and TAZZIOLI [4].

The use of storage rings was first proposed by O'NEILL [5], and an experiment on electron-electron colliding beams is being carried out at Stanford.

A storage ring project like Adone consists roughly of an injector (together with an injection system), a doughnut kept at high vacuum, a magnet and a radiofrequency system. The injector in Adone will be a high-energy linear accelerator for electrons and positrons. Its energy depends on the energy that one wants to attain in the storage ring and must also be large if a large positron intensity is needed. For instance, it is intended in Adone to use a linear accelerator of ~500 MeV for a ring of 750 MeV, and a linear accelerator of ~800 MeV for a ring of 1.5 GeV. The electrons and positrons, after being accelerated by the linear accelerator, are injected, in many pulses, through an optical system into the doughnut. During the accumulation time the magnetic field is kept constant. Then it is varied until the beams reach the desired final energy. The energy variation is provided by the radiofrequency system, whose main purpose is to compensate for the energy losses due to radiation. The stored electrons and positrons circulate on the same orbit in opposite directions and one can look at their annihilation products when they collide. In spite of the high vacuum, collisions also occur with the residual gas in the doughnut, and this effect contributes to produce the "finite life-time" of the beam, which can however be made of the order of ten hours or more by going to relatively high vacuum ($\sim 10^{-9}$ to r).Electron-positron collisions take place only in a few regions of the doughnut, as a result of fact that the electrons and positrons proceed grouped in packets, whose total number is given by the ratio of the radiofrequency to the beam frequency. If the cross-section for a given type of event is σ, the number of events per second that one observes will be $L\sigma$ where L depends on the characteristics of the ring and is called luminosity. The luminosity L is expressed by a simple formula in terms of the parameters of the ring, and it is essentially proportional to the product $s_+ s_-$ of the positron and electron transverse densities (densities on a surface transverse to the beam direction). These densities cannot unfortunately be made arbitrarily high. Their values are limited up to s_{max} which depends on the so-called space-charge effects. In fact, the force acting on one electron (or positron) in the beam is also dependent on the action of the other electrons and positrons. For

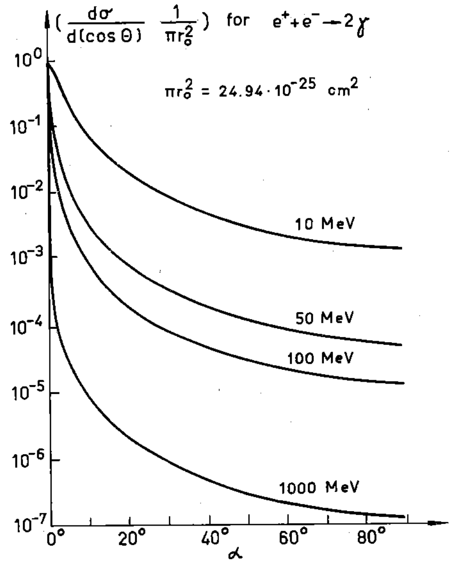

Fig. 1

Perturbation theory cross-section for $e^+ + e^- \to 2\gamma$

and order of magnitude, s_{max} is expected in Adone to be of the order of 10^{11} -10^{12} particles per cm^2 . The cross-sections in perturbation theory decrease rapidly with energy. In Fig. 1, we report the perturbation theory cross-sections for $e^+ + e^- \rightarrow 2\gamma$. In Fig. 2, we report the perturbation theory

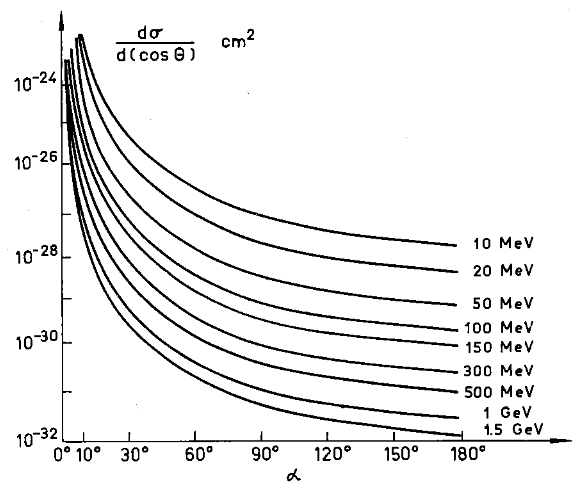

Fig. 2

Differential cross-section for $e^+ + e^- \rightarrow e^+ + e^-$

cross-sections for $e^+ + e^- \rightarrow e^+ + e^-$, and in Fig. 3, we have plotted the perturbation theory cross-section for $e^+ + e^- \rightarrow \mu^+ + \mu^-$, together with the perturbation theory cross-sections for $e^+ + e^- \rightarrow \pi^+ + \pi^-$, $e^+ + e^- \rightarrow p + \bar{p}$, and $e^+ + e^- \rightarrow K + \bar{K}$. The perturbation theory cross-sections for these last three reactions may well be wrong by orders of magnitude, since any effect of strong interactions is entirely neglected. In Fig. 3, the cross-sections are plotted against E/m, where 2E = total centre-of-mass energy and m is the mass of the final particle produced (μ-meson, pion, K-meson, nucleon). In Fig. 4, the perturbation theory cross-section is also reported for the mode of annihilation $e^+ + e^- \rightarrow B + \bar{B}$, where B is a vector meson, on the assumption that it has no anomalous moment. In spite of the general unreliability of perturbation theory estimates, especially when strong interacting particles are produced, it is safe to require that the luminosity L be of the order of $10^{33} cm^2$ per hour in order to have a vast range of experimental uses and a reasonably fast counting rate. To attain such a luminosity one needs a large current intensity. If one defines the intensity as the ratio between the total charge of the beam and the revolution period, the required values are about 100 mA for both positrons and electrons.

POSSIBLE EXPERIMENTS WITH COLLIDING BEAMS OF ELECTRONS AND POSITRONS

Theoretical discussions of electron-positron colliding beam experiments have already been given, A general discussion will not be given in these

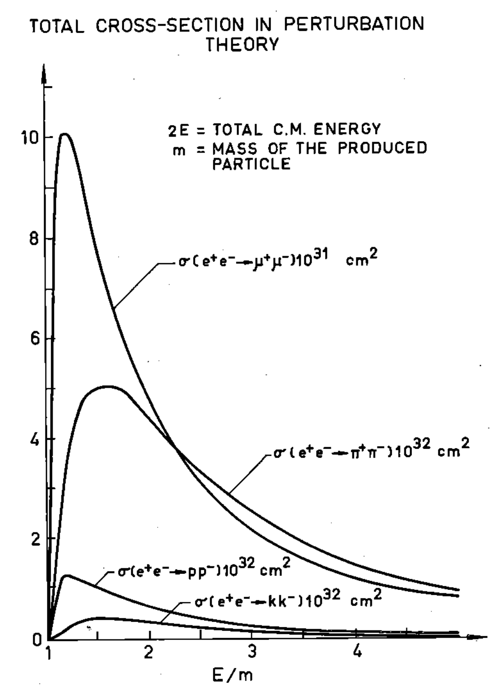

Fig. 3

Total cross-sections in perturbation theory where 2E = total centre-of-mass energy and m = mass of the produced particles

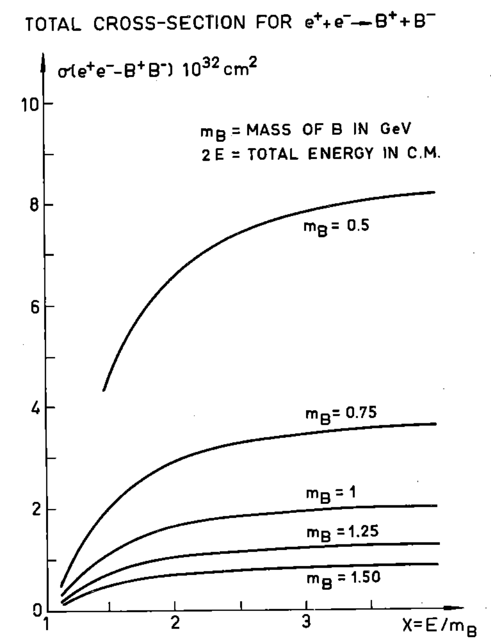

Fig. 4

Total cross-section for $e^+ + e^- \rightarrow B^+ + B^-$ where m_B = mass of B in GeV and 2E = total centre-of-mass energy notes, and I shall refer to previous papers [6]. I shall only mention here a few general points that have been examined

1. Tests of quantum electrodynamics

The reactions $e^+ + e^- \rightarrow 2\gamma$, $e^+ + e^- \rightarrow e^+ + e^-$ and $e^+ + e^- \rightarrow \mu^+ + \mu^-$ can conveniently be used to test quantum electrodynamics at small distances (the last equation will also test the muon structure). A reliable calculation of the radiative corrections is essential for the interpretation of the above reactions. Calculations of the radiative corrections have been carried out at Frascati and at Trieste by Da PRATO, MOSCO and PUTZOLU [7, 8] and by BUDINI and FURLAN [9].

Following the conventional way of modifying electrodynamics one finds that at a beam energy as low as E = 250 MeV (i.e. 500 MeV total centre-of-mass energy) a measurement of $e^+ + e^- \rightarrow 2\gamma$ with a 7% accuracy carried out at 60° - 90° can test electrodynamics up to distances of $\sim$0.2 fermi. A measurement of $e^+ + e^- \rightarrow e^+ + e^-$ around 90° at an energy E $\sim$300 MeV with a 10% accuracy is sufficient to test electrodynamics up to distances of $\sim$0.1 fermi. Similarly, a measurement with a 10% accuracy of $e^+ + e^- \rightarrow \mu^+ + \mu^-$ at E $\sim$300 MeV would also test electrodynamics (or muon structure) to $\sim$0.1 fermi. Tests of electrodynamics can also be conveniently carried out with more complicated reactions such as $e^+ + e^- \rightarrow \mu^+ + \mu^- + \gamma$ which are still easily detectable but have low counting rates.

2. Annihilation into strong interacting particles

The relevant possibility offered by such experiments is that of measuring the electromagnetic form factors of strong interacting particles for time-like values of the momentum transfer. As long as the reaction goes through the one-photon channel, pairs of zero-spin bosons are produced in p states and pairs of spin 1/2 fermions in $^3S_1$ and $^3D_1$ states. As a general remark let me stress one point which is peculiar to e^+ - e^- experiments and makes their interpretation more direct than for the corresponding electron-scattering experiments. In an electron-scattering experiment, such as $e^+ + p^- \rightarrow e + p$, at a given energy the form factors are taken for different values of their argument corresponding to each different scattering angle. On the other hand, in an annihilation experiment such as $e^+ + e^- \rightarrow p + \bar{p}$, at a given energy the form factors are taken at one given value of their argument corresponding to the energy in the experiment. As a result of this, angular distributions are predicted at most up to one parameter (in $e + e \rightarrow p + \bar{p}$ it is the ratio of the magnetic to the charge-form-factor) and there is no need at all for good angular resolutions. I shall not expand here on the topic of annihilation into strong interacting particles, especially as some of the aspects of the discussion will reappear among the special topics I am going to discuss in these notes.

3. Annihilation into pairs of vector mesons

Unless some damping due to rapidly decreasing form factors or to other possible mechanisms occurs, the mode of annihilation

$$e^+ + e^- \to B + \bar{B},$$

where B and $\bar{B}$ are suggested moderately weakly interacting vector bosons, is expected to be a dominant one. Its cross-section in perturbation theory, on the assumption of no anomalous magnetic moment or electric quadrupole moment, is reported in Fig. 4. Comparing this with the other perturbation theory results, one sees that annihilation into vector mesons might well be rather frequent, in spite of the many uncertainties of the electromagnetic properties of this particle. If neutral intermediate vector mesons exist they would also appear as resonances, for instance in $e^+ + e^- \to B^0 \to \mu^+ + \mu^-$. We have calculated that such resonant interaction, if it existed, even after averaging over the experimental energy resolution, could very well compete with the direct electromagnetic process.

For further considerations we again refer to the published papers quoted in reference [6].

In the following I shall only discuss a few points on the subject of $e^+ - e^-$ colliding beams that have been clarified by recent developments. Then I shall talk mostly of $p + \bar{p}$ annihilation experiments.

THE ONE-PHOTON CHANNEL

In this section I shall briefly review the main theoretical notions for the analysis of a reaction

$$e^+ + e^- \to \text{final state} \tag{1}$$

assuming that it mainly proceeds through a graph of the kind shown in Fig. 5.

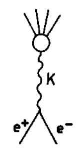

Fig. 5

I shall also discuss some aspects of the problem of radiative corrections to such a graph. The S-matrix element for reaction (1), assumed to proceed through a graph shown in Fig. 5, can be written in the form

$$\langle f | S | e^+ e^- \rangle = \frac{2\pi e}{k^2} (v \gamma_\nu u) \langle f | j_\nu(0) | 0 \rangle \, \delta(q_+ + q_- - q_f) \tag{2}$$

where f denotes the final state of total 4-momentum q_f, $k = q_+ + q_-$ is the 4-momentum of the virtual photon, q_+ and q_- are the e^+ and e^- momentum respectively, v and u are Dirac spinors and j_ν is the electromagnetic current operator. In the centre-of-mass system, which is indeed the laboratory system in colliding beam experiments, one has

$$k^2 = (q_+ + q_-)^2 = -4E^2 \tag{3}$$

where E is the energy of e^+ (or of e^-). The electromagnetic current $j_\nu(x)$ has to satisfy a conservation equation $\delta j_\nu(x)/\delta x_\nu = 0$ from which

$$k_\nu \langle f | j_\nu(0) | 0 \rangle = 0 . \tag{4}$$

In the centre-of-mass system Eq. (4) takes the form

$$k_4 \langle f | j_\nu(0) | 0 \rangle = 2iE \langle f | j_4(0) | 0 \rangle = 0. \tag{5}$$

Thus if we define

$$J_\nu = (2\pi)^{\frac{3n}{2}} \langle f | j_\nu(0) | 0 \rangle \tag{6}$$

where we have introduced a factor $(2\pi)^{\frac{3n}{2}}$ for normalization purposes and where the number of the final particles) we conclude from Eq. (5) that in the centre-of-mass system $J_\nu = (\vec{J}, 0)$ where $\vec{J}$ is a three-dimensional vector. The vector $\vec{J}$ can also be decomposed as $\vec{J} = \vec{J}^{(S)} + \vec{J}^{(V)}$ where the superscripts S and V refer respectively to scalar or vector in isotopic-spin space. It thus follows that the final state must have the following quantum numbers: total angular momentum J = 1, parity P = -1, charge conjugation number C = -1, isotopic spin T = 0 or 1. From the S-matrix element considered in Eq. (2) and from Eq. (6) it follows that the total cross-section for unpolarized initial and final particles is given by

$$\sigma = \frac{(2\pi)^{5-3n}\alpha}{16E^4} \int (d^3 f_1) \ldots (d^3 f_n)\, \delta(E_f - 2E)\, \delta_3(\vec{q}_f)\, T_{mn} \sum_f R_{mn} \tag{7}$$

where $\alpha = e^2/4\pi = 1/137$, f_i is the momentum of the final ith particle, $T_{mn} = 1/2\,(i_m i_n - \delta_{mn})$ where $\vec{i}$ is a unit vector pointing along the direction of the incoming positron and $R_{mn} = -J_m J_n^*$. The summation $\sum_f$ is over the final spin state.

When radiative corrections are included one must add to the graph of Fig. 5 other graphs such as those shown in Figs. 6 and 7.

The graph shown in Fig. 7 cannot be expressed in terms of the vertex indicated indicated in Fig. 8 (suitably modified for the inclusion of radiative corrections).

For the two-photon channel most of the general considerations valid for the one-photon channel (such as its typical selection rules) do not apply. The dominant contribution from the two-photon channel is expected to arise

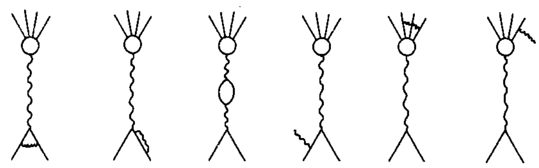

Fig. 6

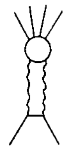

Fig. 7

Fig. 8

from its interference with one-photon graph shown in Fig. 5. This interference is of order e in the cross-section. At this point there is a very simple and general result that can be useful for the interpretation of the experiments. For an experiment which does not distinguish between a final state and its charge conjugate (such as a total cross-section measurement, or any measurement which treats symmetrically the produced charged particles) such an interference term, between the one-photon graph of Fig.5 and the two-photon graph of Fig. 8, does not contribute. Such a theorem is a general consequence of charge-conjugation invariance and can be proved in many ways. A simple proof is the following. Consider a transition from the initial state i to a final state f. We call S_A that part of the S-matrix corresponding to the one-photon channel and S_B that part corresponding to the two-photon channel. The interference term we are considering between the one-photon and the two-photon channel is of the form

$$\mathrm{Re}\sum_f \langle i|S_A|f\rangle\langle f|S_B|i\rangle = \mathrm{Re}\langle i|S_A P_f S_B|i\rangle \tag{8}$$

where $P_f = \sum_f |f\rangle\langle f|$ is a projection operator into those final states f which are selected by the experiment. We assume that the set of states f is invariant under charge conjugation. So we can write

$$\mathcal{C}^{-1} S_A P_f S_B \mathcal{C} = S_A P_f S_B \tag{9}$$

where $\mathcal{C}$ is the charge conjugation operator. Next we note that if we split $|i\rangle$ into a part even under $\mathcal{C}$ and a part odd under $\mathcal{C}$

$$|i\rangle = |i_+\rangle + |i_-\rangle, \tag{10}$$

we have $S_B|i\rangle = S_B|i_+\rangle$ and $S_A|i\rangle = S_A|i_-\rangle$. Thus Eq.(8) can be written as

$$\begin{aligned} \mathrm{Re}\langle i_-|S_A P_f S_B|i_+\rangle &= \mathrm{Re}\langle i_-|\mathscr{C}^{-1} S_A P_f S_B \mathscr{C}|i_+\rangle \\ &= \mathrm{Re}\langle i_-|S_A P_f S_B|i_+\rangle = 0 \end{aligned} \tag{11}$$

which shows the vanishing of such interference term on the assumption that P_f is even under $\mathscr{C}$. Therefore, considerations, valid for the one-photon channel hold including terms e^6, as long as only symmetric experiments are performed.

Still on the subject of radiative corrections there is another feature of electron-positron colliding beams which deserves to be mentioned. The exploration of the limits of validity of quantum electrodynamics can be carried out rigorously only up to energies such that the virtual effects of interacting particles can be neglected. As soon as the breakdown effects to be observed become of the same order as the effects resulting from virtual strong interacting particles a much more complicated theoretical analysis is required, which does not have, so far, the degree of the typical reliability of calculation with electrodynamics. In colliding beam experiments the virtual effects of strong interacting particles come in such a form that they are related to total cross-section measurements carried out with the same colliding beam system. There is therefore, in this respect, no need for a theoretical treatment of strong interactions. Let us consider the modification to the photon propagator arising from virtual strong interacting particles. The modified photon propagator can be written as

$$D^1_{\mu\nu}(k^2) = \frac{\delta_{\mu\nu}}{k^2 - i\epsilon} + \frac{k^2\delta_{\mu\nu} - k_\mu k_\nu}{k^2} \cdot \frac{\bar{\Pi}(0) - \bar{\Pi}(k^2) - i\pi\,\Pi(k^2)}{k^2 - i\epsilon} \tag{12}$$

where

$$\Pi(k^2) = -\frac{(2\pi)^3}{3k^2} \sum_{p_z = k} \langle 0|j_\nu(0)|z\rangle\langle z|j_\nu(0)|0\rangle \tag{13}$$

and

$$\bar{\Pi}(k^2) = P\int_0^\infty \frac{\pi(-a)\,da}{k^2 + a} \,. \tag{14}$$

In Eq.(13), j_ν is the current operator and the sum is extended over all physical states with total four-momentum $p_z = k$. The photon propagator in Eq.(12) is defined as

$$D^1_{\mu\nu}(x - x') = i\langle 0|P[A_\mu(x)A_\nu(x')]|0\rangle \tag{15}$$

where P is the chronological product and A_μ is the electromagnetic field. Formally Eq. (12) cannot be written as

$$D^1_{\mu\nu}(k^2) = \frac{\delta_{\mu\nu}}{k^2} + \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\int_0^\infty \frac{da\,\Pi(-a)}{a(k^2+a-i\epsilon)}. \quad (16)$$

It is easy to see that the experimentally measured cross-sections for processes $e^+ + e^- \rightarrow \gamma \rightarrow F$ where F denotes a group of final states, is directly related to the contribution to Eq. (13) from the group of state F among the states on which the summation is extended. In this way one can calculate the modifications of the photon propagator resulting from virtual strong interacting particles, directly from measured values of cross-sections.

Let us note that in Eq. (13) the matrix elements $\langle 0|j_\nu(0)|z\rangle$ of the current j_ν are proportional to J_ν as defined from Eq. (6). In fact, the total cross-section for annihilations leading to the final states F, can be written, in the centre-of-mass system, as

$$\sigma_F(E) = -\frac{(2\pi)^5\alpha}{16E^4} T_{mn} \sum_{p_z = k} \langle 0|j_m(0)|z\rangle\langle z|j_n(0)|0\rangle. \quad (17)$$

This is again Eq. (7) in a different notation. The sum occuring in Eq. (13) is slightly different from that in Eq. (17). In Eq. (13) there occurs a scalar product of the two matrix elements of j_ν, while in Eq. (17) the two spaces indices m and n are different. However one can use gauge invariance to relate the two expressions. One has

$$(2\pi)^3 \sum_{F,\, p_z = k} \langle 0|j_\mu(0)|z\rangle\langle z|j_\nu(0)|0\rangle = \Pi_F(k^2)(k_\mu k_\nu - k^2\delta_{\mu\nu}) \quad (18)$$

where $\Pi_F(k^2)$ denotes the contribution to $\Pi(k^2)$ from the group of intermediate states F. If we substitute Eq. (18) into Eq. (17), we obtain the direct connection we were looking for, namely

$$\sigma_F(E) = (\pi^2\alpha/E^2)\Pi_F(-4E^2). \quad (19)$$

In Eq. (16) for $D^1_{\mu\nu}(k^2)$ we can now substitute $\Pi_F(-4E^2)$ from Eq. (19) to get a formal expression for the photon propagator modified for the virtual contribution of the states F

$$D_{\mu\nu}(k) = \frac{\delta_{\mu\nu}}{k^2} + \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k}\right)\frac{2}{\pi^2\alpha}\int_0^\infty \frac{EdE\,\sigma_F(E)}{k^2+4E^2-i\epsilon}. \quad (20)$$

In Eq. (20) σ_F is at least of order α^2 so that the modifications are usually very small. In some exceptional cases, however, they become rather big. Such a situation occurs, for instance, at energies corresponding to the ω^0 mass. The cross-section $\sigma_F(E)$ for processes that go through a ω^0 intermediate state is very sharply peaked around the ω^0 mass. The processes that can go through ω^0 are those in which the final isotopic spin is zero, apart from isotopic-spin mixing effects that are indeed quite large (see the discussion of such resonant contributions in the next section). The effect of virtual ω brings about noticeable modifications also in processes such as $e^+ + e^- \to e^+ + e^-$ or $e^+ + e^- \to \mu^+ + \mu^-$ in which no strong interacting particles are present in both initial and final state. Such effects become difficult to detect in experiments with rough energy resolution, but they become very apparent as soon as the energy resolution is of the same order as the lifetime of the resonant state. Such a situation is probably realizable with $e^+ - e^-$ colliding beams, where it should be possible to reach energy resolutions of the order of 100 keV without very great technical difficulties [10, 11] .

THE RESONANT CONTRIBUTION FROM ρ^0 AND ω^0

We have seen that states with J = 1, P = -1, C = -1, T = 0 or 1, can be coupled to $e^+ e^-$ in the one-photon channel. The vector mesons ρ^0 and ω^0 that have recently been discovered have the right quantum numbers to couple in the one-photon channel. They may give rise to resonant contributions through graphs of the general kind as shown in Fig. 9.

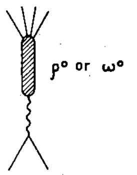

Fig. 9

Recently there has been some information about the couplings of ρ^0 and ω^0. One is thus able to estimate the cross-sections for processes going through intermediate ρ^0 and ω^0. I shall first review here the general strategy that one follows to determine the couplings of ρ^0 and ω^0. Of course much of the work is based on theoretical models (mainly on the so-called ρ-dominant model by Gell-Mann) and therefore some of the conclusion may well turn out to be not very accurate. We believe, however, that the qualitative features of such an approach are essentially right and that the main conclusions that we derive (such as the almost unbelievably large values of the cross-sections through intermediate ω^0) are essentially independent of the model. In reference [12] are listed some of the papers that are relevant for the problem of the determination of the couplings of ρ^0 and ω^0. I shall now describe, without going into details, the strategy that leads to a determination of the coupling constants.

(1) $\omega^0 \to \pi^+ + \pi^- + \pi^0$ decay.

In the ρ-dominant model one describes such a decay through the graph shown in Fig. 10. One needs therefore two coupling constants: $f_{\omega\rho\pi}$ and $\gamma_{\rho\pi\pi}$.

(2) $\rho \to \pi + \pi$ decay

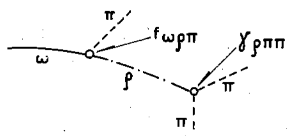

Fig. 10

The width Γ for the decay which is shown in Fig. 11, is expressed in terms of $\gamma_{\rho\pi\pi}$. Inserting for Γ a value $\sim$100 MeV one gets

$$\gamma_{\rho\pi\pi} / 4\pi \simeq 1/2 \ . \tag{21}$$

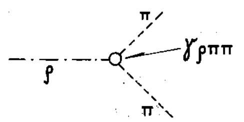

Fig. 11

(3) Pion form factor

The γ - ρ coupling constant is called $em_\rho^2 / 2\gamma_\rho$, where m_ρ is the ρ-mass. To determine γ_ρ , one writes down the expression for the pion form as given in terms of the graph shown in Fig. 12:

$$eF(k^2) = (em_\rho^2 / \gamma_\rho)\ [1/(m_\rho^2 + k^2)]\ \gamma_{\rho\pi\pi} \ .$$

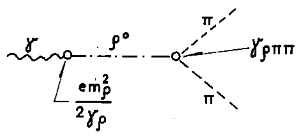

Fig. 12

From F(0) = 1, one gets

$$\gamma_{\rho\pi\pi} / \gamma_\rho = 1 \ . \tag{22}$$

From (21) and (22) one has

$$\gamma_\rho^2 / 4\pi \simeq 1/2. \tag{23}$$

It should be noted that one would also derive the relation $\gamma_{\rho NN} / \gamma_\rho = 1$ from the nucleon isovector form factor, by a similar procedure. In general, as stressed by Gell-Mann and Sakurai, it is required that the coupling of ρ be universal: $\gamma_{\rho\pi\pi} = \gamma_{\rho NN} = \gamma_{\rho KK}$.

(4) $\omega \to \pi + \gamma$ decay

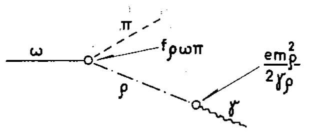

Fig. 13

For this decay mode (Fig. 13), one still needs $f_{\omega\rho\pi}$. γ_ρ is known by Eq.(23). However, the branching ratio between $\omega \to \pi^0 + \gamma$ and $\omega \to 3\pi$ can already be predicted at this stage.

(5) $\pi^0 \to \gamma + \gamma$ decay

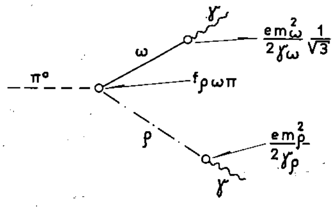

Fig. 14

The coupling constant γ - ω is defined as $em_\omega^2/2\gamma_\omega\sqrt{3}$ (see Fig. 14). One needs γ_ω and $f_{\omega\rho\pi}$. From unitary symmetry one has

$$\gamma_\omega = \gamma_\rho \ . \tag{24}$$

This is the reason why the factor $\sqrt{3}$ was inserted in the definition of the ω - γ coupling constant. Then inserting a value for the π^0 lifetime one determines $f_{\omega\rho\pi}$. Inserting $\Gamma_\pi \sim 3$ eV, one finds $f_{\omega\rho\pi}$ and, from such a value and from Eq. (21), one derives

$$\Gamma_\omega \simeq 0.4 \text{ MeV}. \tag{25}$$

The determination of the coupling constants is now complete. For instance, one can compute the rate for $\omega \to \pi + \pi$ through the decay shown in Fig. 15., for $\omega \to e^+ + e^-$ (or $\mu^+ + \mu^-$) through the decay shown in Fig. 16

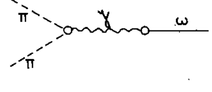

Fig. 15

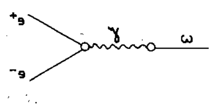

Fig. 16

and for similar decay modes for ρ. We recall that two main assumptions have been made: the ρ-dominant model and unitary symmetry, in obtaining Eq. (24). The colliding-beam experiments, that I am going to describe, will provide a check of such assumptions.

Let us first consider the contribution from ρ^0 intermediate states. The reaction $e^+ + e^- \to \pi^+ + \pi^-$ can go through ρ^0 excitation and consequent decay. The resonant cross-section is

$$\sigma_{r+s} \simeq 3.4 \times 10^{-31}\,\mathrm{cm}^2 . \tag{26}$$

To compare it with a typical colliding-beam cross-section, we consider $e^+ + e^- \to \mu^+ + \mu^-$, the cross-section of which is

$$\sigma \cong (1/3)\,\alpha^2 \pi \lambda\!\!\!^{-2}, \tag{27}$$

as soon as the energy is such that one can neglect the μ-mass. The cross-section (26) is about 1.7 times larger than that for annihilation into 2μ at the same energy.

One may also wonder whether the contribution of intermediate ρ^0 states can become important in the electromagnetic processes $e^+ + e^- \to e^+ + e^-$ and $e^+ + e^- \to \mu^+ + \mu^-$. In fact, by a rough estimate one can convince oneself that this will not be the case. A detailed calculation was carried out by BROWN and COLOGERO showing that the effects are negligible [13].

Now let us consider the contribution from ω^0 intermediate states. The very important feature here is the very narrow width of ω^0. We have estimated that it should be of the order of 0.4 MeV, according to Eq. (25). The energy resolution with $e^+ + e^-$ colliding beams may easily be pushed down to a small fraction of a MeV. Under such conditions the ω^0 will show up in a very apparent way, and its will in fact originate the dominant contributions to the cross-sections at energies around its rest mass. We consider the following three processes

$$e^+ + e^- \to \omega^0 \to \pi^+ + \pi^- + \pi^0, \tag{28}$$

$$e^+ + e^- \to \omega^0 \to \pi^0 + \gamma, \tag{29}$$

$$e^+ + e^- \to \omega^0 \to \pi^+ + \pi^-, \tag{30}$$

all proceeding through intermediate ω^0. Making use of the information derived before on the couplings of ω^0, we get for the total cross-sections of the reactions (28), (29) and (30), at the resonant energy:

$$\sigma_r(3\pi) = 7 \times 10^{-29}\,\mathrm{cm}^2, \tag{31}$$

$$\sigma_r(\pi^0\gamma) = 4 \times 10^{-30}\,\mathrm{cm}^2, \tag{32}$$

$$\sigma_r(2\pi) = 0.8 \times 10^{-30}\,\mathrm{cm}^2. \tag{33}$$

These values of the cross-sections are almost unbelievably large. We compare them with the cross-section for $e^+ + e^- \to \mu^+ + \mu^-$ as given by the formula $\sigma(2\mu) \simeq 1/3\,\pi\alpha^2\lambda\!\!\!^{-2}$. Note however that this formula does not hold at an energy near the ω^0 mass, just because there is also an amplitude arising from $e^+ + e^- \to \omega^0 \to \mu^+ + \mu^-$ which interferes with the purely electrodynamic

namic amplitude for $e^+ + e^- \to \mu^+ + \mu^-$, as discussed in the previous section. These two amplitudes are of the same order of magnitude. We find that: $\sigma_r(3\pi)$, as given by Eq. (31), is 39.0 times larger than $1/3\,\pi\alpha^2\lambda\!\!\!^{-2}$; $\sigma_r(\pi^0\gamma)$ is 22 times larger; $\sigma_r(2\pi)$ is 4.7 times larger. It may also be relevant to stress that $e^+ + e^- \to \pi^0 + \gamma$ is of higher electromagnetic order than $e^+ + e^- \to \mu^+ + \mu^-$. In fact, the processes (28), (29) and (30) are of order e^4, e^6 and e^8 respectively. The decay of ω^0 in (29) occurs in fact by violating isotopic-spin selection rules since the isospin of ω is T = 0 and that of the final state must be T = 1. This illustrates how peculiar the situation becomes at that energy. Again, we want to stress the interest of an observation of the interference effect of $e^+ + e^- \to \omega^0 \to \mu^+ + \mu^-$ with the lowest order electromagnetic amplitude.

PROTON-ANTIPROTON ANNIHILATION INTO LEPTONS AND INTO VECTOR MESONS

I shall talk now on an experiment which is closely related to the colliding beam experiments we have been discussing, and which is being carried out at CERN by CONVERSI, FARLEY, MÜLLER and ZICHICHI [14]. The experiment consists in measuring the cross-section for:

$$p + \bar{p} \to e^+ + e^-. \tag{34}$$

The possibility of measuring the cross-sections for

$$p + \bar{p} \to \mu^+ + \mu^- \tag{35}$$

and

$$p + \bar{p} \to B + \bar{B}, \tag{36}$$

where B is the suggested intermediate vector boson, is also being examined by the same group. These experiments were discussed in detail in a paper by ZICHICHI, BERMAN, CABIBBO and GATTO [15].

Let us first discuss the $p + \bar{p} \to e^+ + e^-$ process. It goes through the graph shown in Fig. 17 and is essentially the inverse process of $e^+ + e^- \to$

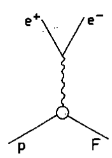

Fig. 17

$p + \bar{p}$. Any consideration valid for $e^+ + e^- \to p + \bar{p}$ also applies, with the relevant changes, to $p + \bar{p} \to e^+ + e^-$. The proton electromagnetic vertex is explored in the experiment for timelike values of the momentum transfer. In the equal notations the proton electromagnetic vertex is given by

$$\bar{u}(p)\{F_1(k^2)\gamma_\mu + [F_2(k^2)/2M]\sigma_{\mu\nu}\, k_\nu\}\, u(p) \tag{37}$$

where we have defined

$$\sigma_{\mu\nu} = (1/2)(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu),$$
$$F_1(0) = e$$
$$F_2(0) = e\mu_p .$$

In Fig. 18 we consider the real k^2 axis where $k = q_+ + q_-$ is the virtual-photon momentum, and indicate the physical region of the electron-proton scattering experiments ($k^2 > 0$), the physical region for $e^+ + e^- \rightarrow p + \bar{p}$ or $p + \bar{p} \rightarrow e^+ + e^-$ ($k^2 < -4M^2$), and the absorptive region ($k^2 < -4\mu^2$). The form factors are complex only in the absorptive region.

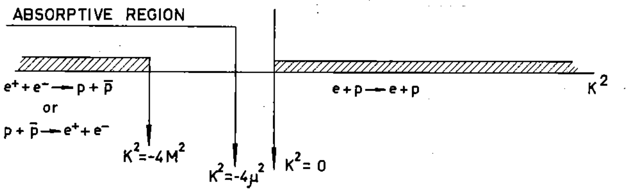

Fig. 18

The cross-section for the process (34) is given in the centre-of-mass system by

$$\frac{d\sigma(p\bar{p}\rightarrow e^+e^-)}{d(\cos\theta_c)} = \frac{\pi}{8}\,\frac{\alpha^2}{E\sqrt{E^2-M^2}}\left[|F_1+F_2|^2(1+\cos^2\theta_c) + \left|\frac{M}{E}F_1 + \frac{E}{M}F_2\right|^2 \sin\theta_c\right] \tag{38}$$

where F_1 and F_2 are taken at $k^2 = -4E^2$, E being the $\bar{p}$ energy in the centre-of-mass system and θ_c the centre-of-mass angle. With the form factors $G_1 = F_1 + F_2$ and $G_2 = F_1 + (E/M)^2 F_2$, Eq. (38) can be written as

$$\frac{d\sigma(p\bar{p}\rightarrow e^+e^-)}{d(\cos\theta_c)} = \frac{\pi}{8}\,\frac{\alpha^2}{E\sqrt{E^2-M^2}}\left[|G_1|^2(1+\cos\theta_c) + \left(\frac{M}{E}\right)^2|G_2|^2\sin\theta_c\right]. \tag{39}$$

Numerical estimates of what the cross-section might be have very little value since nothing is known on the form factors in the timelike region. For example, we can consider two models:

(i) pointlike proton model

In this case $F_1 = e$ and $F_2 = 1.79\, e$;

(ii) extrapolation of a fit to spacelike experiments [16]

In this model, we have the following form factors:

$F_1 = e[1 - 1.18\, k^2/(k^2 + 30\mu^2)]$ and $F_2 = 1.79\, e\, [1 - 1.59\, k^2/(k^2 + 30\mu^2)]$.

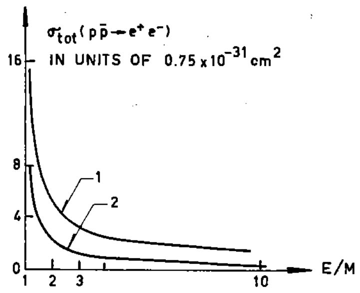

Fig. 19

Total cross-section for $p + \bar{p} \to e^+ + e^-$ in units of 0.75×10^{-31} cm^2

In Fig. 19 we have reported the values which one obtains in this way for σ_{tot} $(p\bar{p} \to e^+e^-)$, in units of 0.75×10 cm^2. The upper curve 1 is for point-like proton with $\mu_p = 1.79$, the lower curve 2 is for model (ii).

With luck, from the angular distribution, one may measure $|G_1|^2$ and $|G_2|^2$. However, in the physical region of the experiment the form factors are complex. This fact produces a polarization effect that does not occur in electron-proton scattering, in the physical region of which the form factors are real. Suppose you can dispose of polarized antiproton beams. If they collide on unpolarized protons and annihilate into $e^+ + e^-$, the cross-section is not the same for all φ, but there is an azimuthal effect, depending on the cosine of the angle that the normal to the production plane forms with the antiproton polarization vector. This effect is proportional, of course, to the sine of the phase difference between the two form factors F_1 and F_2 (or G_1 and G_2). The cross-section is given by

$$\frac{d\sigma}{d(\cos\theta_c)} = \left.\frac{d\sigma}{d(\cos\theta_c)}\right|_{unp} + \frac{M}{E}\,\mathrm{Im}\,(G_1^* G_2)\,|\sin 2\theta_c|\,(\vec{P}\cdot\vec{n}) \qquad (40)$$

where $\vec{P}$ is the antiproton polarization vector and $\vec{n}$ the normal to the production plane. If the experiment is, instead, carried out with unpolarized antiprotons on a polarized-proton target, Eq. (40) holds again, except for changing the + sign into a - sign (this follows in general from the TCP theorem). If we write

$$G_1 = |G_1|\,e^{i\delta_1}\ ; \qquad (41)$$

$$G_2 = |G_2|\,e^{i\delta_2}\ , \qquad (42)$$

the experiment would inform us on $\sin(\delta_1 - \delta_2)$. However, some independent knowledge on the phases can be gained by the use of dispersion relations. It is clear that dispersion relations, giving an equation connecting the real part to the imaginary part of a form factor, can also be interpreted as giving a connection of the phase to the modulus. In fact an equation, such as

$$\operatorname{Re} G(k^2) = \frac{1}{\pi} P \int_{4\mu^2}^{\infty} \frac{dm^2 \operatorname{Im} G(m^2)}{(m^2 + k^2)} , \tag{43}$$

can also be interpreted as an integral equation for the phase $\delta(k^2)$, once $|G(k^2)|$ is known:

$$|G(k^2)| \cos\delta(k^2) = \frac{1}{\pi} P \int_{4\mu^2}^{\infty} dm^2 \frac{|G(m^2)| \sin\delta(m^2)}{(m^2 + k^2)} . \tag{44}$$

Eq. (44) has inconvenient mathematical features. A most convenient way of using such an information has been suggested by FREUND and KUMMER [17]. Consider a function $f(z) = |f(z)| \exp[i\delta(z)]$ which (a) is analytic in the cut plane z (Fig. 20), the cut going from a to ∞; (b) has no complex zeros; and (c) satisfies $f(z^*) = f^*(z)$.

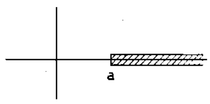

Fig. 20

Suppose you know $|f(z)|$ on the whole real axis. Because of the condition (c), $|f(z)|$ has no discontinuity; in fact $|f(x+i\epsilon)| - |f(x-i\epsilon)| = |f(x+i\epsilon)| - |f^*(x+i\epsilon)| = 0$.

What can be said on $\delta(z)$? To this purpose we consider $\log + (z) = \log |f(z)| + i\delta(z)$. We use a subtracted Cauchy relation (z_0 is the point of subtraction), taking the contour indicated in Fig. 21. We have explicitly

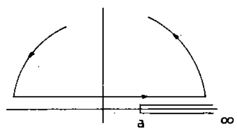

Fig. 21

used our restriction (b) on the zeros of f(z). Taking now z and z_0 real, and assuming that the integral on the circle can be neglected, we obtain

$$\log f(x) - \log f(x_0) = \frac{x - x_0}{2\pi i} \int_{-\infty}^{+\infty} \frac{\log f(x')\, dx'}{(x' + i\epsilon - x)(x' + i\epsilon - x_0)} .$$

Now we use the identity

$$(x - x_0) \frac{1}{(x' - x + i\epsilon)} \cdot \frac{1}{(x' - x_0 + i\epsilon)} = (x - x_0) P \frac{1}{(x' - x)} \cdot \frac{1}{(x' - x_0)}$$

and, by separating the real part from the imaginary part, we find

$$\delta(x) - \delta(x_0) = -\frac{x - x_0}{2\pi} P \int_{-\infty}^{+\infty} \frac{\log |f(x')| dx'}{(x' - x)(x' - x_0)} \tag{45}$$

which is our desired equation, and also

$$\log |f(x)| - \log |f(x_0)| = \frac{x - x_0}{2\pi} P \int_{Q}^{+\infty} \frac{\delta(x')\, dx'}{(x' - x)(x' - x_0)} \tag{46}$$

Let us apply Eq. (45) to $G_1 = |G_1| \exp(i\delta_1)$. It is convenient to subtract at $k^2 = 0$. We obtain

$$\delta_2(k^2) = -\frac{k^2}{2\pi} P \int_{-\infty}^{+\infty} \frac{\log |G_2(-m^2)| dm^2}{(k^2 + m^2)\, m^2} \,. \tag{47}$$

How can one use Eq. (47)? Let us split the integration region into three parts:

$$\int_{-\infty}^{+\infty} = \int_{-\infty}^{-4M^2} + \int_{-4M^2}^{0} + \int_{0}^{+\infty}$$

In the first region one can insert the measured values of $G_2(k^2)$ (from $p + \bar{p} \rightarrow e^+ + e^-$ or $e^+ + e^- \rightarrow p + \bar{p}$). In the second one can only make guesses; fortunately resonances seem to dominate in that region. In the third region one again inserts experimental values (from $e + p \rightarrow e + p$). The guesses are not quite arbitrary. In fact one must find $\delta(k^2)$ such that: (a) it must be real away from the absorptive region, i. e. for all $k^2 - 4\mu^2$; (b) $\delta_2(k^2) - \delta_2(k^2)$ must fit the values measured in the polarization experiment described before.

In the expressions (38) and (39) terms of the order $(m_e/M)^2$, where m_e is the electron mass and M is the nucleon mass have safely been neglected. If one wants to take into account the muon mass in the process $p + \bar{p} \rightarrow \mu^+ + \mu^-$, one has simply to introduce a factor β_μ (velocity of final μ in the centre-of-mass system) in front of Eq. (38) or (39) and make the following substitution

$$(1 + \cos^2\theta_c) \rightarrow (2 - \beta_\mu^2 \sin^2\theta_c)\,,$$

$$\sin^2\theta_c \rightarrow (1 - \beta_\mu^2 \cos^2\theta_c)\,.$$

For the total cross-sections one then finds

$$\frac{\sigma_t\,(p\bar{p} \rightarrow \mu\mu)}{\sigma_t\,(p\bar{p} \rightarrow e\,e)} = \frac{1}{2}\,\beta_\mu\,(3 - \beta_\mu^2) = 1 - \left(\frac{3}{8}\right)\left(\frac{m_\mu}{E}\right)^4 + 0\left(\left(\frac{m_\mu}{E}\right)^6\right). \tag{48}$$

This branching ratio is almost exactly equal to one. More important than these kinematic corrections are the radiative corrections. Their calculation is being carried out at Frascati. For the evaluation of them, the remark we made before concerning the interference effects between the two-photon and the one-photon channels comes out to be very relevant and simplifying. The 2μ versus 2e ratio in $p\bar{p}$ annihilation offers a very suitable way of measuring a possible high-energy breakdown of electrodynamics or a possible muon structure. The momentum transfers are always larger than 2M and they are the largest so far considered in such experiments. Furthermore, they are time-like and thus they provide essentially different information that provided space-like experiments.

The last topic I am going to review here is the mode of annihilation $p+\bar{p} \rightarrow B+\bar{B}$ where B is a vector meson as suggested for mediating weak interactions.

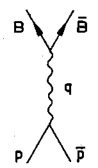

Fig. 22

The process occurs through graph shown in Fig. 22. The most general form of the electromagnetic vertex, for a spin-one boson is, on invariance grounds

$$J_\mu = G_2 (\epsilon_1 \epsilon_2) p_\mu + (G_1 + \mu G_2 + \epsilon G_3) [(\epsilon_1 q) \epsilon_{2\mu} - (\epsilon_2 q) \epsilon_{1\mu}]$$

$$+ \epsilon G_3 m_B^{-2} [(q\epsilon_1)(q\epsilon_2) - 1/2\, q^2 (\epsilon_1 \epsilon_2)] p_\mu \qquad (49)$$

where p is the difference of the final four-momenta of B and $\bar{B}$, ϵ_1 and ϵ_2 are the polarization vectors of B and $\bar{B}$, m_B is the mass of B, $\mu+\epsilon$ is a possible anomalous magnetic moment of B and 2ϵ a possible anomalous electric quadrupole moment. The form factors G_1, G_2 and G_3 depend on the squared momentum transfer q^2.

We also define the bilinear combinations

$$R = (1/2) |G_1 + \mu G_2 + \epsilon G_3|^2 (E/m_B)^2,$$

$$S = (1/2) |G_1 + 2(E/m_B)^2 \epsilon G_3|^2 + (1/4) G_1 + 2(E/m_B)^2 \mu G_2|^2.$$

The general expression for the cross-section is given, in the centre-of-mass system,

$$\frac{d\delta(p\bar{p} \rightarrow B\bar{B})}{d(\cos\theta)} = \frac{\pi\alpha^2}{2EP} \beta_B^3 \left\{ R(A+B) + SA + (S-R)(B-A)\cos^2\theta \right\}, \qquad (50)$$

$$\delta_T (B\bar{B}) = \frac{\pi\alpha^2}{3EP} \beta_B^3 (2A+B)(2R+S). \qquad (51)$$

In Eqs. (50) and (51) β_B is the velocity of B; $A = (1/2)|F_1 + F_2|^2$ and $B = (1/2) |(M/E) F_1 + (E/M) F_2|^2$ are exactly the same combinations of the nucleon form factors which determine the angular distribution of $p+\bar{p} \rightarrow e^+ + e^-$.

Similarly, $2A + B$ also determines the total cross-section for $p + \bar{p} \rightarrow e^{+} + e^{-}$. One thus finds for the ratio of $B\bar{B}$ annihilation to $e^{+}e^{-}$ annihilation

$$b = \sigma_t\,(p\bar{p} \rightarrow B\bar{B})/\sigma_t\,(p\bar{p} \rightarrow e^{+}e^{-}) = \beta_B^3\,(2R + S). \tag{52}$$

Eq. (52) holds in the most general case, and is still valid if the antiprotons are at rest.

If B has no anomalous moments and constant form factors, b is simply: $b = \beta_B^3\,[(3/4) + (E/m_B)^2]$. In Fig. 23 this branching ratio is reported versus E/m_B. Of course E must always be larger than the nucleon mass. One sees that annihilation into a pair of intermediate mesons is favoured with respect to annihilation into $e^{+}e^{-}$ or $\mu^{+}\mu^{-}$ already for a centre-of-mass energy larger than $1.5\,m_B$, provided B has no anomalous electromagnetic properties. In Fig. 23 we have also reported b for $\mu = +1$ and $\mu = -1$, $\epsilon = 0$ and constant form factors. Once B is produced according to Eq. (36) it will

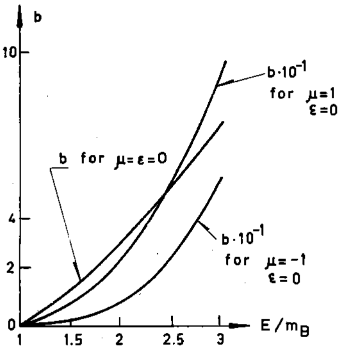

Fig. 23

Branching ratio versus E/m_B

decay rapidly (in about 10^{-17}s) into its disintegration products (2π, 3π, $\pi + K$, $\mu + \nu$, $e + \nu$, etc.). The annihilation events will exhibit definite angular correlations and in some cases they will be of the kind:

$$p + \bar{p} \rightarrow B^{+} + B^{-} \rightarrow (\mu + \nu) + (\pi + \pi),$$

$$p + \bar{p} \rightarrow B^{+} + B^{-} \rightarrow (\mu + \nu) + (e + \nu),$$

$$p + \bar{p} \rightarrow B^{+} + B^{-} \rightarrow (K^0 + \pi^{+}) + (\pi^{-} + \pi^0) \text{ etc.},$$

which should allow the identification of B. Branching ratios among the various decay modes of B have recently been discussed by BERNSTEIN and FEINBERG [18].

To show that Eq. (49) is the most general form of the vertex J_μ we note that there are four independent four-vectors, out of which J_μ must be formed. They are $q = p_1 + p_2$, $p = p_1 - p_2$, ϵ_1 and ϵ_2, where p_1 and p_2 are the final momenta of B and $\bar{B}$. It can easily be checked that

$$p^2 + q^2 = -4\,m_B^2$$

and so, one can form the following independent scalars

$$q^2, \ (\epsilon_1 \cdot k), \ (\epsilon_2 \cdot k), \ \text{and} \ (\epsilon_1 \cdot \epsilon_2).$$

Then, observing that J_μ must be linear in both ϵ_1 and ϵ_2, one can write in general:

$$\begin{aligned} J_\mu = & \ q_\mu \left[a(k^2)(\epsilon_1\epsilon_2) + b(k^2)(\epsilon_1 k)(\epsilon_2 k) \right] \\ & + p_\mu \left[c(q^2)(\epsilon_1\epsilon_2) + d(q^2)(\epsilon_1 q)(\epsilon_2 q) \right] \\ & + \epsilon_{1\mu}(\epsilon_2 k)\, e(k^2) \ + \epsilon_{2\mu} \ (\epsilon_1 k)\, f(k^2). \end{aligned}$$

One has now to impose:

$$k_\mu J_\mu = 0$$

which gives

$$Q(k^2) = 0,$$

$$b(k^2)k^2 + c(k^2) + f(k^2) = 0.$$

However, $J_\mu \to -J_\mu$ when $p \to -p$ and $\epsilon_1 \to \epsilon_2$ (because of charge conjugation invariance). Therefore $e = -f$ and consequently also $b = 0$. Using all these conditions and by a redefinition of the form factors one gets the general expression (49) given above.

REFERENCES

[1] TOUSCHEK, B., Proceedings of the Conference on High Energies, CERN (1961).

[2] BERNARDINI, C., CORAZZA, G.F., GHIGO, G. and TOUSCHEK, B., Nuovo Cimento 18 (1960) 1293.

[3] AMMAN, F., BERNARDINI, C., GATTO, R., GHIGO, G. and TOUSCHEK, B., LNF 61/5, Frascati National Laboratories, (January 27, 1961).

[4] AMMAN, F., BASSETTI, M., BERNARDINI, M., CORAZZA, G.F., MASSAROTTI, A., MANGO, L., PELLEGRINI, C., PLACIDI, M., PUGLISI, M., TAZZIOLI, F., in Ricerca Scientifica (to be published).

[5] O'NEILL, G.K., Proceedings of the International Conference on High Energy Accelerator and Instrumentation, CERN (1959) 125.

[6] GATTO, R., Proceedings of the Conference on High Energy Physics, CERN(1961); CABIBBO, N. and GATTO, R., Phys. Rev. Lett. 4 (1960), 313; Phys. Rev. 124 (1961) 1577; GATTO, R, Proceedings of the Aix-en-Provence Conference on Elementary Particles (1961).

[7] PUTZOLU, G., Nuovo Cimento 20 (1961) 542.

[8] DA PRATO, MOSCO and PUTZOLU (to be published)

[9] BUDINI and FURLAN, Phys. Rev. (to be published)

[10] DRELL, S., Annals of Physics 4 (1958) 75.

[11] KÄLLEN, G., Helv. Phys. Acta 25 (1952) 417.

[12] GELL-MANN, M. and ZACHARIASEN, F., Phys. Rev. 124 (1961) 953; NAMBU, Y. and SAKURAI, Phys. Rev. Lett. 7 (1961) 469; GELL-MANN, SHARP and WAGNER, Phys. Rev. Lett. 8 (1962) 261; GELL-MANN, M., Phys. Rev. 125 (1962) 1067.

[13] BROWN, L.M. and COLOGERO, F., Phys. Rev. 120 (1960) 653.

[14] CONVERSI, FARLEY, MÜLLER and ZICHICHI (private communication).

[15] ZICHICHI, BERMAN, CABIBBO and GATTO, Nuovo Cimento 24 (1962) 170.

[16] FUBINI, S., Proceedings of the Aix-en-Provence Conference on Elementary Particles (1961).

[17] FREUND, P. and KUMMER, Nuovo Cimento 24 (1962) 1160.

[18] BERNSTEIN, J. and FEINBERG, G., Proceedings of the Aix-en-Provence Conference on Elementary Particles (1961).